NECT

FET MATHEMATICS

TRAINING HANDOUT

TERM 1 & 2 2019

CONTENTS

MATHEMATICS
is not about
numbers, equations,
computations, or
algorithms:
it is about
UNDERSTANDING.

William Paul Thurston

TRAINING PROGRAMME

TERM 1 & 2 2019

	TIME	ACTIVITY	TEACHER WORKSHOP
1	30 minutes	Welcome, housekeeping and updates Introductions, reflections and agenda	
2	30 minutes	Pre-training Activity	
3	30 minutes	Orientation to materials and feedback from Term 3 & 4	
4	1 hour	Conceptual understanding in a learning centred classroom – the basics and principles	
5	2 hours	Patterns: Grade 10 - 12	
6	2 hours 30 minutes	Trigonometry: Grade 10 & 12	
7	2 hours	Functions and Inverses: Grade 12	
8a	30 minutes	Preparation for lesson demonstrations	
8b	1 hour	Preparation for lesson demonstrations	
9	2 hours 30 minutes	Lesson demonstrations and feedback	
10	1 hour	Assessment	
11	1 hour	Orientation to the trainer's guide	
12	1 hour	Training of teachers: planning session	
13	30 minutes	Post test	
14	30 minutes	Final questions and answers Closure and evaluation	

CONCEPTUAL UNDERSTANDING IN A LEARNING CENTRED CLASSROOM

INTRODUCTION

- Procedural fluency
- Strategic competence
- Mathematical reasoning
- Conceptual understanding

Up to now, much attention was spent on procedural fluency. The focus of our work in Term 1 & 2 this year is on conceptual understanding and the learning centred classroom – two concepts that need to be blended, as it is suggested that conceptual understanding is created in a learning centred classroom.

CONCEPTUAL UNDERSTANDING

Your own understanding of a concept:

Your own understanding of conceptual understanding:

5 tips to assist in a conceptual understanding:

- Belief
- Sense Making
- Scaffolding
- Time
- Multiple Representations

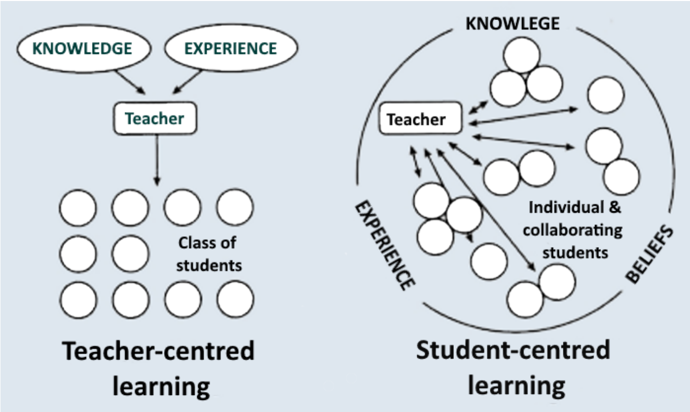
Notes:

DIFFERENT ORIENTATIONS TOWARDS TEACHING

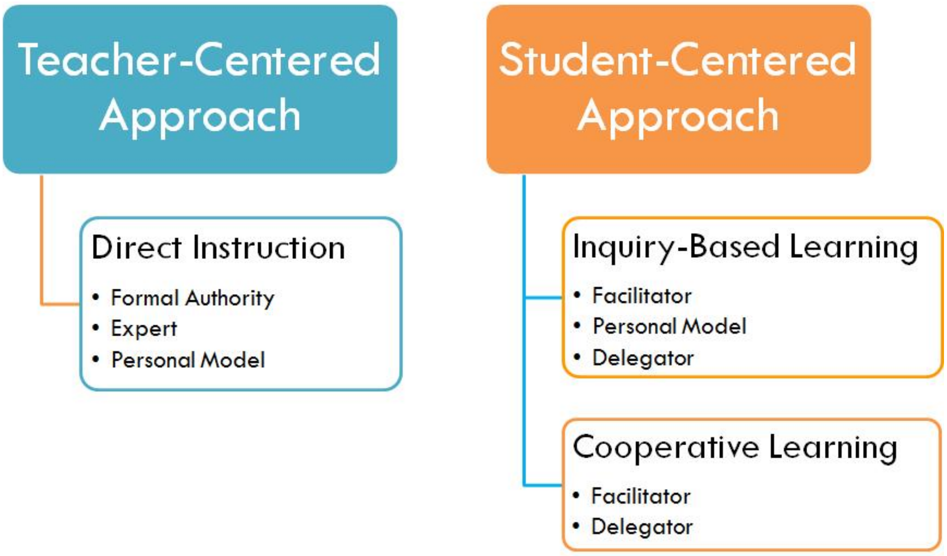
Three main orientations towards teaching include a) the teacher centred classroom, b) the learner centred classroom and c) the learning centred classroom.

1. Teacher-centred classroom
2. Learner-centred classroom
3. Learning-centred classroom

Teacher-centred vs learner-centred



https://lo.unisa.edu.au/mod/book/view.php?id=610988&chapterid=102030



https://www.plaz-tech.com/technology-in-the-classroom-making-the-shift-from-teacher-centered-to-student-centered-approach/

Notes:

Learning centred classroom

The learning centred classroom has a few characteristics and requirements:

They are:

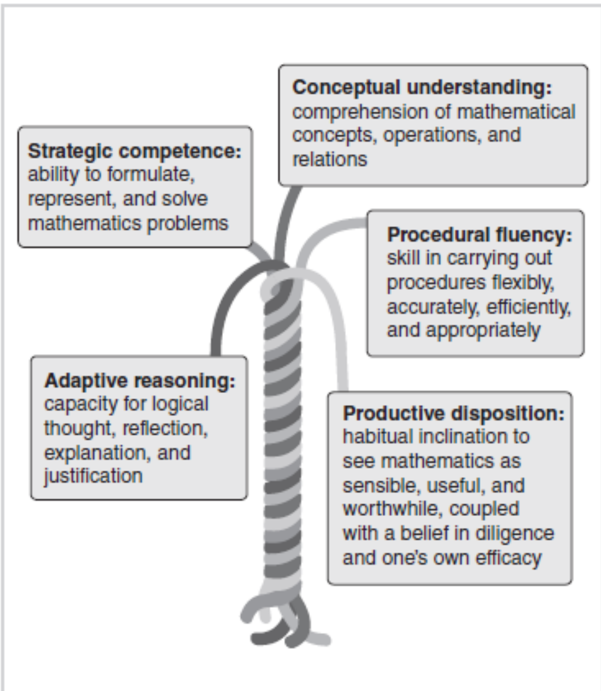
- specific
- intentional
- semi-structured
- meaningful
- reciprocal
- realistic
- transcended
- assessed
- reasonable
- solidified

Notes:

According to Daniel Willingham, successful mathematics learning requires three different abilities that must be developed and woven together. These are:

- control of facts
- control of processes
- conceptual understanding

Notes:



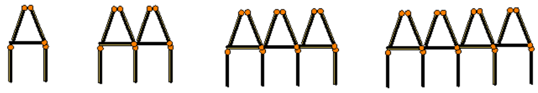
Source: Reprinted with permission from Kilpatrick, J., Swafford, J., & Findell, B. (Eds.), *Adding It Up: Helping Children Learn Mathematics.* Copyright 2001 by the National Academy of Sciences. Courtesy of the National Academies Press, Washington, D.C.

PATTERNS: GRADE 10 – 12

Using the function $y = 4x + 1$, complete the following table and plot the points on a cartesian plane.

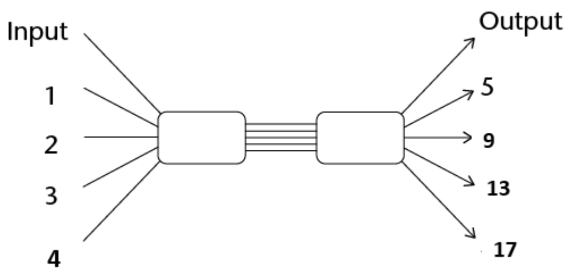
x	1	2	3	4
y				

The following pattern is made with matchsticks:



Find the general term to represent the number of matches used in each pattern.

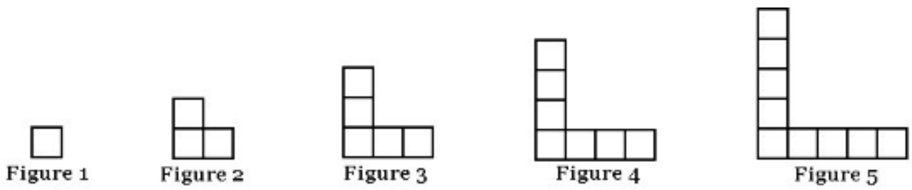
Find the rule in the following flow chart and make up a word problem to match the values given.



Using the function $y = 2x - 1$, complete the following table and plot the points on a cartesian plane.

x	1	2	3	4
y				

The following pattern is made from squares:



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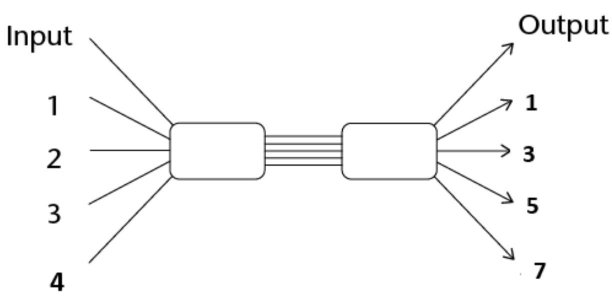
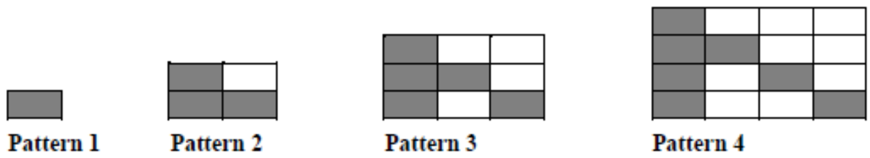


Example:

Dark tiles (D) and light tiles (L) are used to create patterns on a floor. The first four patterns are shown below. For the patterns that follow the tiles are arranged in a similar manner.



a) How many dark tiles were used in pattern 5?

b) How many light tiles were used in pattern 6?

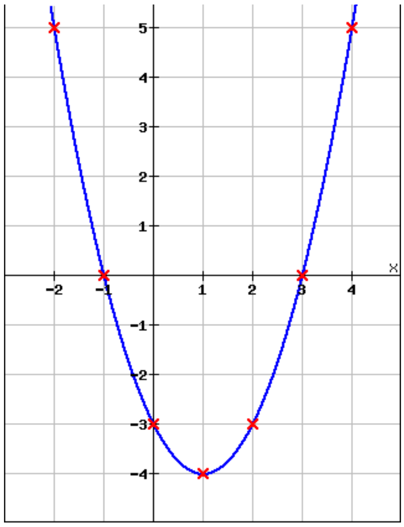
c) Write down the general term (D_n) for the number of dark floor tiles used in each pattern.

d) Write down the general term (L_n) for the number of light floor tiles used in each pattern.

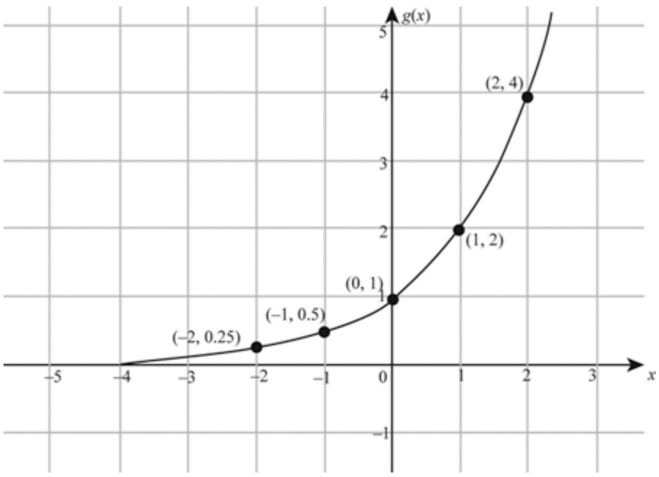
e) Which pattern will have exactly 64 light floor tiles?

NOV 2016

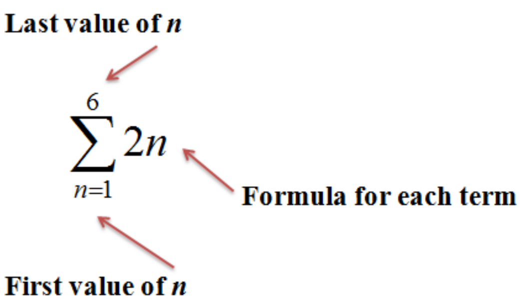
Quadratic patterns



Geometric patterns



Sigma notation



TRIGONOMETRY: Gr 10 & 12

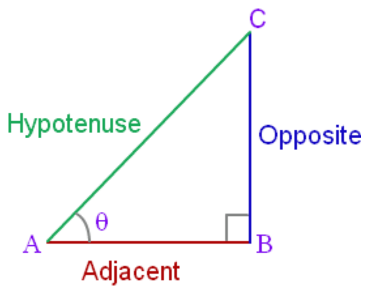
INVESTIGATION - Introduction to Trigonometry

ACTIVITY 1 – Naming right-angled triangles

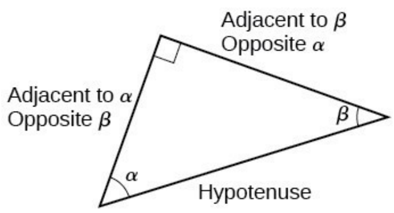
An important aspect of gaining an understanding of the basic principles of trigonometry is to know how to accurately label the sides of a right-angled triangle.

The hypotenuse should be the easiest one for you – it is always across from the right angle. One could see it as the right angle marking is pointing at the hypotenuse.

The other two sides are called the ‘adjacent’ and the ‘opposite’, as shown below:



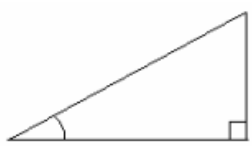
How do we know which to use? They are named according to the angle of interest (we will call this the reference angle). This is demonstrated below:



Note that ‘adjacent’ is always next to the angle and ‘opposite’ is always across from the angle.

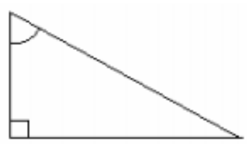
Label the following sides according to the angle marked:

1.

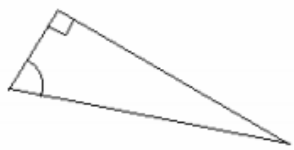


2.

3.



4.



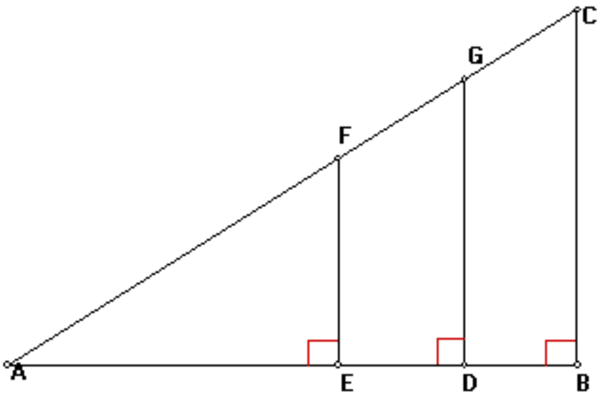
ACTIVITY 2 – Similar triangles

In Grade 9 you learned about similar triangles. Similar triangles have two properties:

- All angles are equal
- The sides are in proportion

(Stop now to discuss this with your teacher if you don't remember what this means)

Look carefully at the diagram below. You should notice that there are three right-angled triangles.



The first (and smallest) one has been named for you. Name the other two:

$\Delta\ AEF$		

All three triangles have a right angle and all three triangles share the angle A. This means the angle A is common to all of them.

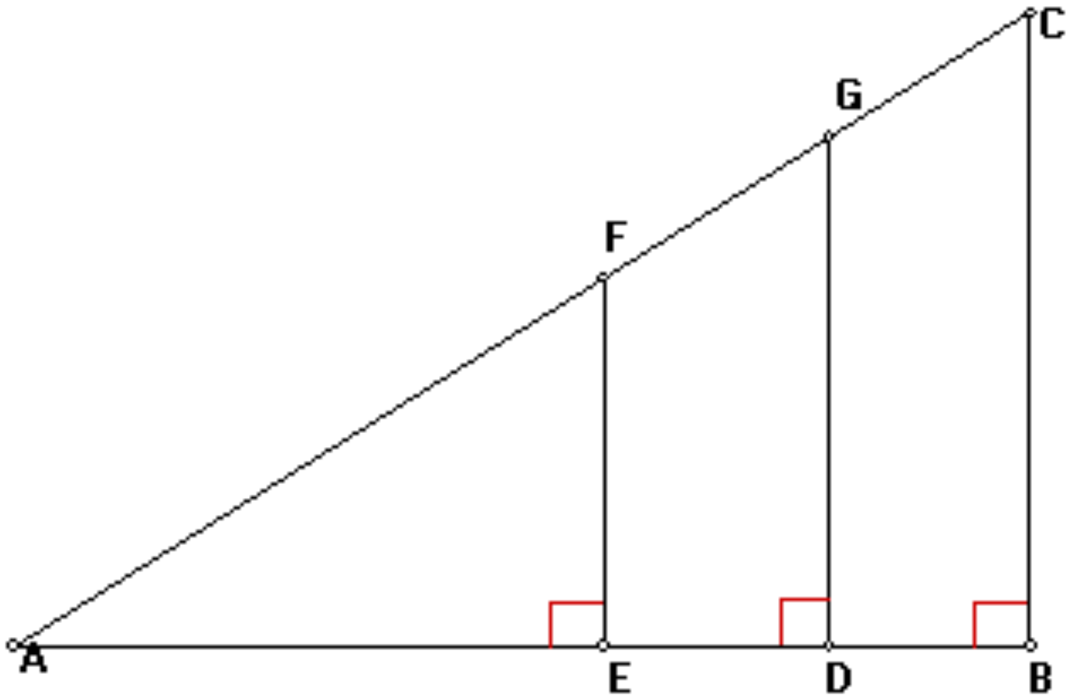
What can be said about the third angle in each of the three triangles?

What geometry theorem did you use to make this statement?

Important idea: These three triangles are similar to each other. This means that their sides are in proportion.

ACTIVITY 3 – Investigating ratios

Using the diagram below, answer the questions that follow. Accuracy is important – be careful when measuring.



1. Using a protractor, measure the angle A to the nearest whole degree. Confirm that it is 32^o.
2. Label the sides of $\Delta\, AEF$ according to $\hat{A}$.
3. Using a ruler, measure the sides of $\Delta\, AEF, \Delta\, ADG$ and $\Delta\, ABC$ and complete the table below. Write your answer to the nearest millimetre.

<table>
<tr><td rowspan="9">Reference angle ($\hat{A}$):
32^o</td><td colspan="3">$\Delta\, \boldsymbol{AEF}$</td></tr>
<tr><td>Opposite (EF)</td><td>Adjacent (AE)</td><td>Hypotenuse (AF)</td></tr>
<tr><td>mm</td><td>mm</td><td>mm</td></tr>
<tr><td colspan="3">$\Delta\, \boldsymbol{ADG}$</td></tr>
<tr><td>Opposite (DG)</td><td>Adjacent (AD)</td><td>Hypotenuse (AG)</td></tr>
<tr><td>mm</td><td>mm</td><td>mm</td></tr>
<tr><td colspan="3">$\Delta\, \boldsymbol{ABC}$</td></tr>
<tr><td>Opposite (BC)</td><td>Adjacent (AB)</td><td>Hypotenuse (AC)</td></tr>
<tr><td>mm</td><td>mm</td><td>mm</td></tr>
</table>

Use these measurements to complete the following table. Use a calculator to simplify the ratios found and round your answer to THREE decimal places.

	$\Delta\ \boldsymbol{AEF}$	$\Delta\ \boldsymbol{ADG}$	$\Delta\ \boldsymbol{ABC}$
Reference angle $(\hat{A})$	$\frac{EF}{AF} = \frac{}{} =$	$\frac{DG}{AG} = \frac{}{} =$	$\frac{BC}{AC} = \frac{}{} =$
32^o	$\frac{AE}{AF} = \frac{}{} =$	$\frac{AD}{AG} = \frac{}{} =$	$\frac{AB}{AC} = \frac{}{} =$
	$\frac{EF}{AE} = \frac{}{} =$	$\frac{DG}{AD} = \frac{}{} =$	$\frac{BC}{AB} = \frac{}{} =$

What do you notice?

Using the top row as a reference and the names given to the sides of a triangle, what sides were used to find the ratio?

Using the middle row as a reference and the names given to the sides of a triangle, what sides were used to find the ratio?

Using the bottom row as a reference and the names given to the sides of a triangle, what sides were used to find the ratio?

Using your calculator, calculate the following and round your answers to three decimal places.

sin $32^o =$	cos $32^o =$	tan $32^o =$

ACTIVITY 4 – Why does trigonometry work the way it does?

Key concepts:

When you put sine 32^o into the calculator, the solution given is the **ratio** of what the length of the opposite side divided by the length of the hypotenuse would give you in ANY right-angled triangle in which the reference angle is 32^o.

When you put cosine 32^o into the calculator, the solution given is the **ratio** of what the length of the adjacent side divided by the length of the hypotenuse would give you in ANY right-angled triangle in which the reference angle is 32^o.

When you put tangent 32^o into the calculator, the solution given is the **ratio** of what the length of the opposite side divided by the length of the adjacent side would give you in ANY right-angled triangle in which the reference angle is 32^o.

It is important that you understand that sine, cosine or tangent of ANY ANGLE will always be a RATIO.

Using your calculator, find $\tan 45^o$ ____________

Explain in words what you have found:

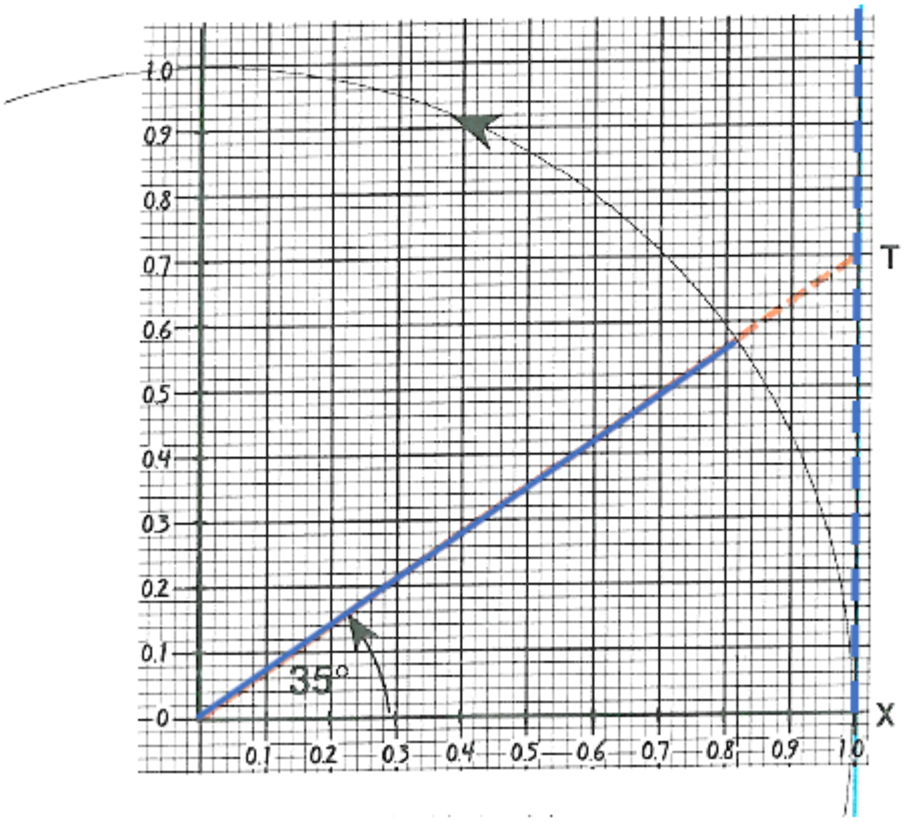
The ratio of the ___________________ side divided by the ____________________ side in a right-angled triangle where the reference angle is ________

What type of triangle is this other than a right-angled one? _____________________

Explain why this ratio is equal to the whole number that it is and not a decimal like the previous answers found ____

More information on the tangent of an angle and why it is called this:

Consider this diagram:



The darker line starting at the origin and forming the angle 35^o is the radius of the part circle visible and is **one unit** long. Confirm this by noting where it begins on the x –axis as well as where it intersects with the y –axis.

It has rotated anti-clockwise from the x –axis to form the angle of 35^o at the origin.

The dashed vertical line $(x = 1)$, only touches the circle at one place. That place is the point $(1; 0)$.

A line that only touches a circle at one point is called a TANGENT.

The darker line extended meets this tangent formed at T. $XT = 0{,}7$ units.

Find $tan35^o$ using your calculator ___________ 

COMPOUND AND DOUBLE ANGLES

Investigation – Trigonometry

PART 1

Using a scientific calculator, complete the following table. Write your answer to THREE decimal places. Answers only are acceptable.

Let $A = 20^o$ and $B = 10^o$	
$\cos(A + B)$	
$\cos A + \cos B$	
$\cos(A - B)$	
$\cos A - \cos B$	
$\cos A . \cos B + \sin A . \sin B$	
$\cos A . \cos B - \sin A . \sin B$	
Let $A = 50^o$ and $B = 30^o$	
$\sin(A + B)$	
$\sin A + \sin B$	
$\sin(A - B)$	
$\sin A - \sin B$	
$\sin A . \cos B + \cos A . \sin B$	
$\sin A . \cos B - \cos A . \sin B$	

Look carefully at your answers on the right-hand side and make conclusions about the expressions on the left-hand side. Complete the following statements by filling in $=$ or $\neq$ in the space provided.

$\text{Cos}(A + B)$ ☐ $\cos A + \cos B$	$\sin(A + B)$ ☐ $\sin A + \sin B$
$\cos(A - B)$ ☐ $\cos A - \cos B$	$\sin(A - B)$ ☐ $\sin A - \sin B$
$\cos(A + B)$ ☐ $\cos A . \cos B - \sin A . \sin B$	$\sin(A + B)$ ☐ $\sin A . \cos B + \cos A . \sin B$
$\cos(A - B)$ ☐ $\cos A . \cos B + \sin A . \sin B$	$\sin(A - B)$ ☐ $\sin A . \cos B - \cos A . \sin B$

For those statements that you have concluded are equal, choose two different values for A and B to confirm that you are correct.

Statement that seems to be true:	
New values tested: $A =$ $B =$	$LHS =$ $RHS =$
Conclusion:	The statement above is true/false
Statement that seems to be true:	
New values tested: $A =$ $B =$	$LHS =$ $RHS =$
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Statement that seems to be true:	
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Conclusion:	The statement above is true/false
Statement that seems to be true:	
New values tested: $A =$ $B =$	$LHS =$ $RHS =$
Conclusion:	The statement above is true/false

Using a scientific calculator, complete the following table. Write your answer to THREE decimal places. Answers only are acceptable.

Let $A = 20^o$	
$\sin 2A$	
$2\sin A.\cos A$	
Let $A = 50^o$	
$\sin 2A$	
$2\sin A.\cos A$	

Before completing the next table, remember that $cos^2 10^o = (\cos 10^o)^2$. Be careful when using your calculator.

Let $A = 20^o$	
$\cos 2A$	
$2cos^2 A - 1$	
$1 - 2sin^2 A$	
$cos^2 A - sin^2 A$	
Let $A = 50^o$	
$\cos 2A$	
$2cos^2 A - 1$	
$1 - 2sin^2 A$	
$cos^2 A - sin^2 A$	

The table above should show that:

$\sin 2A = 2 \sin A . \cos A$	$\cos 2A = 2cos^2 A - 1$ $\cos 2A = 1 - 2sin^2 A$ $cos^2 A - sin^2 A$

Complete the following:

$sin^2 A + cos^2 A =$
$cos(90^o - A) =$
$sin(90^o - A) =$
$cos(90^o + A) =$
$sin(90^o + A) =$

These will be used in a later lesson to derive all the identities that you have proved to be true.

Example 1

If $cos\ \theta = k$, and $\theta \epsilon [0^0; 90^0]$, determine the value of $sin\left(\frac{\theta}{2} + 30^0\right).cos\left(\frac{\theta}{2} + 30^0\right)$ with the aid of a diagram and without the use of a calculator.

Example 2:

Find the general solution of:

$$\cos 2\theta - \cos\theta + 1 = 0$$

Example 3:

Prove the identity:

$$\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$$

LESSON DEMONSTRATIONS

Grade 10	• Number Patterns • Trigonometry (any one aspect)
Grade 11	• Number Patterns • Trigonometry (any one aspect)
Grade 12	• Functions and Inverses • Geometric Sequences • Series (arithmetic or geometric) • Sigma notation • Sum to infinity

The presentations must include:

Introduction (in which 'learners' are reminded of where we finished off yesterday and what they already know)

Direct Instruction (in which the next concept in the topic will be taught)

The lesson must be taught exactly as it would be in the classroom. Do not tell your colleagues HOW you would teach the lesson – the lesson must be taught as if you are in the classroom and your colleagues are the learners.

Each member of the group must be in front and assisting in some way.

A division of labour is recommended. For example, someone can prepare the 'board work' on flipchart paper, someone can present the introduction, and someone can do the actual direct instruction. If there are many worked examples to be done, these can also be split amongst participants (i.e.: the 'teacher' can change during the presentation).

REFLECTING ON THE LESSONS THAT YOU TEACH

It is important to reflect on your teaching. Through reflection, we become aware of what is working and what is not, what we need to change and what we do not. Reflecting on your use of these lesson plans will also help you use them more effectively and efficiently.

These lesson plans have been designed to help you deliver the content and skills associated with CAPS. For this reason, it is very important that you stick to the format and flow of the lessons. CAPS requires a lot of content and skills to be covered – this makes preparation and following the lesson structure very important.

Use the tool below to help you reflect on the lessons that you teach. You do not need to use this for every lesson that you each – but it is a good idea to use it a few times when you start to use these lessons. This way, you can make sure that you are on track and that you and your learners are getting the most out of the lessons.

LESSON REFLECTION TOOL	
Preparation*	
1.	What preparation was done?
2.	Was preparation sufficient?
3.	What could have been done better?
4.	Were all of the necessary resources available?

Classroom Management			
		Yes	**No**
5.	Was information written on the board?		
6.	Were examples written on the board?		
7.	Was the solution to examples discussed with the learners in a meaningful way?		
8.	Overall reflection on this part of the lesson: What was done well? What could have been done better?		
Accessing Information			
		Yes	**No**
9.	Was something written on the chalkboard before the lesson started?		
10.	Was the work on the board neat and easy for the learners to read?		
11.	Was the explanation on the content easy to follow?		
12.	Was the information on the board used effectively to help with the explanations?		
13.	Was any new vocabulary taught effectively?		
14.	Were the learners actively engaged? (asked questions, asked for their opinions and to give ideas or suggestions)		
15.	Overall reflection on this part of the lesson: What was done well? What could have been done better?		

Conceptual Development			
		Yes	**No**
17.	Was the information taught in the 'Accessing Information' part of the lesson used to foreground the activity?		
18.	Were clear instructions given for the conceptual development activity?		
19.	Were the outcomes/answers to the activities explained to the learners?		
20.	Could the learners ask questions and were explanations given?		
21.	Was a model answer supplied to the learners? (written or drawn on the board)		
21.	Were the checklist questions used effectively?		
22.	At the end of the lesson, were the learners asked if they had questions or if they needed any explanations?		
23.	Overall reflection on this part of the lesson: What was done well? What could have been done better?		

ASSESSMENT

The types of assessment required according to CAPS

CAPS specify that four types of assessment should be used. These are:

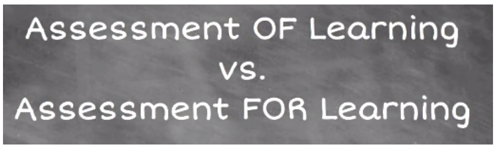
- Baseline assessment
- Diagnostic assessment
- Formative assessment
- Summative assessment

Notes:

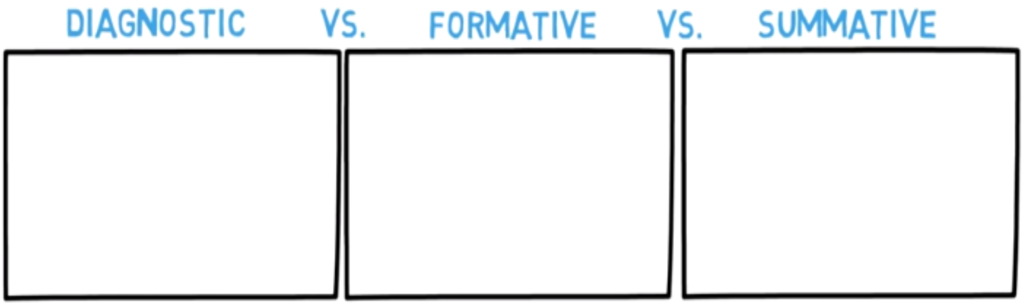
Video clips

Assessment For Learning vs. Assessment Of Learning

Notes:

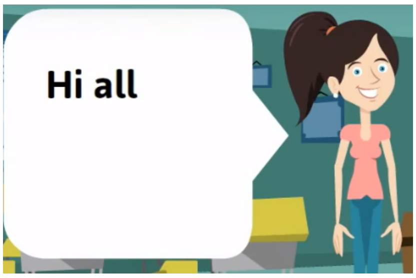


Formative vs. Summative vs. Diagnostic Assessment



Notes:

Assessment for Learning



Notes:

All educational actions must support students' learning of more and better mathematics; assessment is no exception.

Resources

Number patterns resource

Using the function $y = 4x + 1$, complete the following table and plot the points on a cartesian plane.

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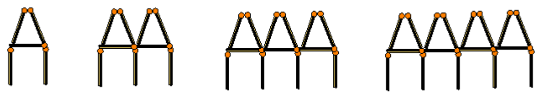

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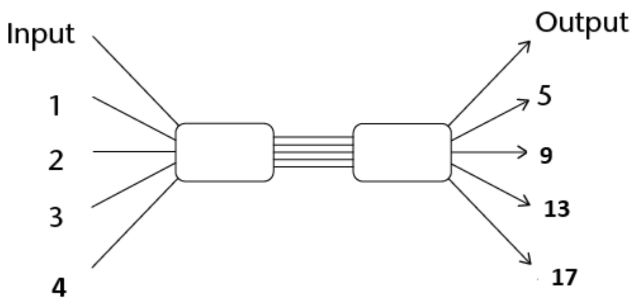
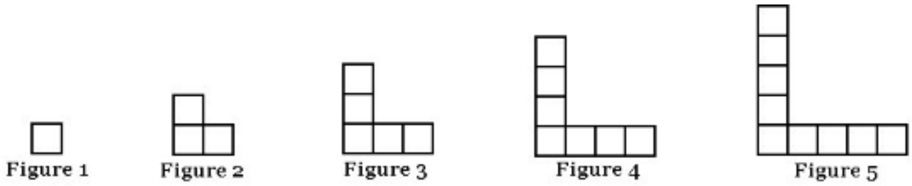


Using the function $y = 2x - 1$, complete the following table and plot the points on a cartesian plane.

x	1	2	3	4
y				

The following pattern is made from squares:



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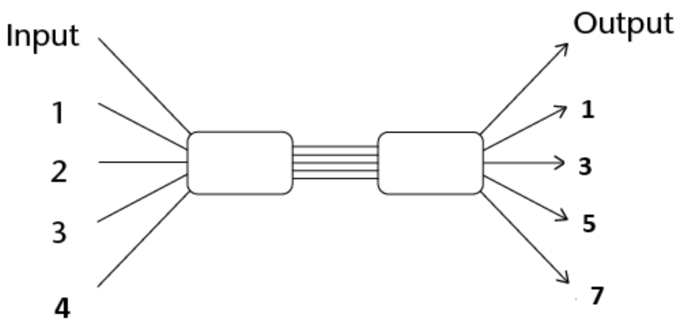
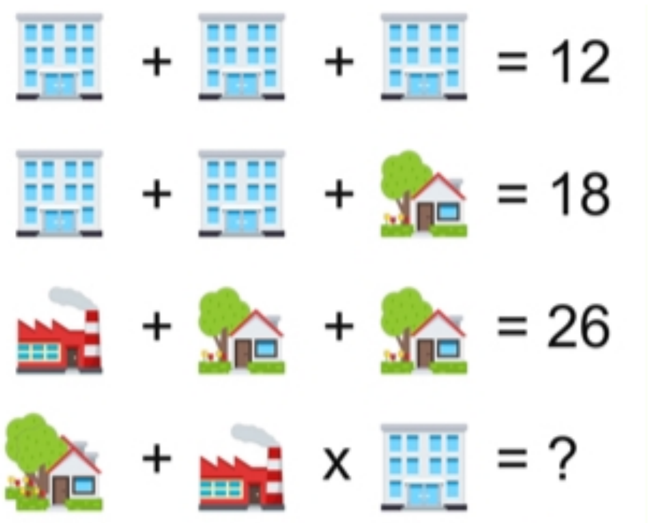
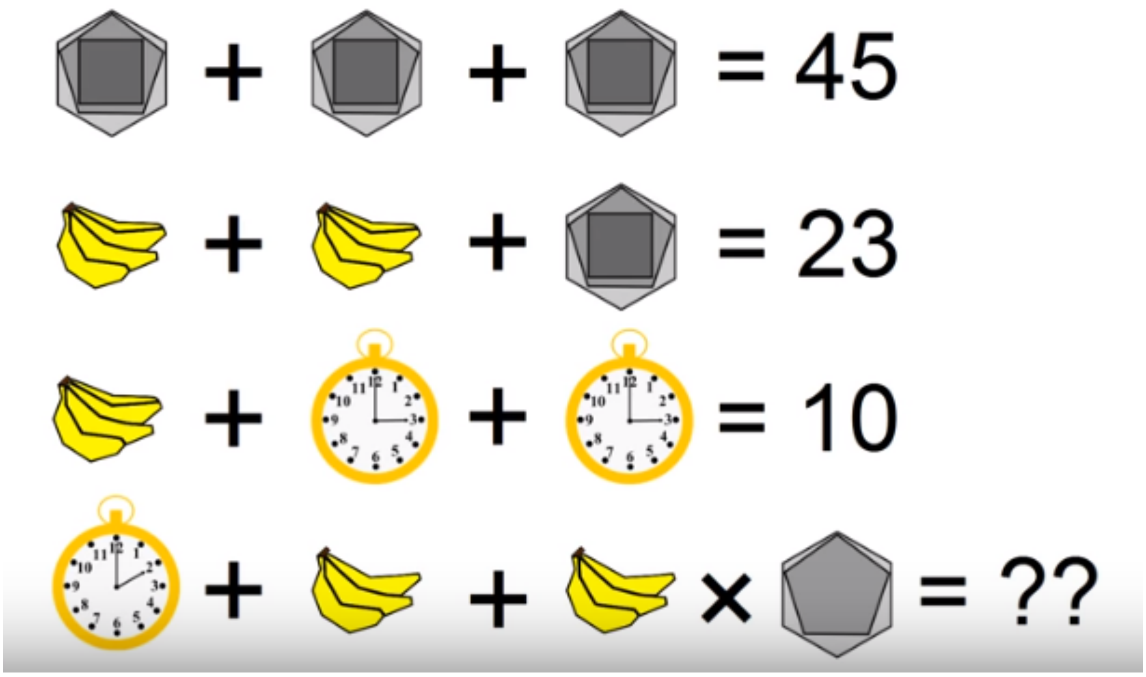
Find the rule in the following flow chart and make up a word problem to match the values given.




+ + = 12
+ + = 18
+ + = 26
+ x = ?


+ + = 45
+ + = 23
+ + = 10
+ + × = ??

Trigonometry resource

Compound angles and Double angles

Deriving the compound and double angle formulae is required for examination purposes.

To do this, we accept the identity $\cos(A-B) = \cos A.\cos B + \sin A.\sin B$ and use it as the starting point.

(Note: To prove $\cos(A-B) = \cos A.\cos B + \sin A.\sin B$, the unit circle, distance formula and the cosine rule are used. This proof is not required.)

Other useful information to be used is:

$sin^2A + cos^2A =$	1
As well as: $sin^2A = 1 - cos^2A$ $cos^2A = 1 - sin^2A$	
$cos(90^o - A) =$	$\sin A$
$sin(90^o - A) =$	$\cos A$
$cos(90^o + A) =$	$-\sin A$
$sin(90^o + A) =$	$\cos A$
$\sin(-A) =$	$-\sin A$
$\cos(-A) =$	$\cos A$

Starting in the centre of the following page and following the arrows, complete the proofs to derive each successive identity.

The key is to use the identity where the arrow starts from. Rewrite the one required in the format of the preceding identity then use information from above to rewrite it further until the required identity has been derived.

$$\cos(A - B) = \cos A.\cos B + \sin A.\sin B$$

1

$\cos(A + B) =$

____

____

____

∴ **cos** $(A + B) =$ ______________________

2

$\cos(2A) =$

____

____

____

∴ **cos** $(2A) =$ ______________________

3

$\cos(2A) =$

____

____

____

∴ **cos** $(2A) =$ __________

4

$\cos(2A) =$

____

____

____

∴ **cos** $(2A) =$ ____________

5

$\sin(A + B) =$

____

____

____

____

∴ **sin** $(A + B) =$ ______________________

6

$\sin(2A) =$ ____________

____________________ = ____________________

∴ **sin** $(2A) =$ ______________________

7

$\sin(A - B) =$

____

____

____

____

∴ **sin** $(A - B) =$ ______________________